A MONTE CARLO STUDY ON THE DYNAMICAL FLUCTUATIONS INSIDE QUARK AND ANTIQUARK JETS[1]

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ABSTRACT

The dynamical fluctuations inside the quark and antiquark jets are studied using Monte Carlo method. Quark and antiquark jets are identified from the 2-jet events in e^+e^- collisions at 91.2 GeV by checking them at parton level. It is found that transition point exists inside both of these two kinds of jets. At this point the jets are circular in the transverse plane with respect to the property of dynamical fluctuations. The results are consistent with the fact that the third jet (gluon jet) was historically first discovered in e^+e^- collisions in the energy region 17-30 GeV.

Keywords: Quark jet, Antiquark jet, Dynamical fluctuations

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1 Introduction

As is well known, in high energy collisions the hadronization of partons, being a soft process, can not be analyzed using perturbative QCD (pQCD). Therefore, phenomenological analysis has to be applied to study the properties of the final state hadronic system, in order to get information which would lead to a better understanding of the dynamics of strong interaction. Investigating the dynamical fluctuations inside jets produced by the fragmentation of partons is an effective way to explore the dynamics in high energy collisions.

In a 2-jet event in e^+e^- collisions, the two produced jets are expected to be back to back and the thrust (or sphericity) axis coincides with the jet axes. Furthermore, the two jets being produced from a quark and an antiquark respectively are expected to have similar property. Under these assumptions the study of the dynamical fluctuations inside a single jet could be performed through the investigation of the fluctuations in a 2-jet event. Recently, this kind of investigation has been carried out both theoretically [1] and experimentally [2].

In view of the importance of this problem, it is worthwhile studying the dynamical fluctuations inside a single jet directly [3]. The aim of this paper is to carry on this study for quark and antiquark jets separately using Monte Carlo method. Through this study, the influence of the above mentioned two assumptions are gotten rid of and the dynamical fluctuation properties in the hadronization process of quark and antiquark can be directly obtained.

2 Selection of quark and antiquark jets

Event samples of e^+e^- collision at c.m. energy $\sqrt{s} = 91.2$ GeV are produced using LUND JETSET7.4 Monte Carlo code. Only charged particles are used in the calculation.

Two-jet events are selected using the Durham jet algorithm[4]. In this scheme, a jet resolution variable $y_{\rm cut}$ is defined for every pair of particles (or jets) i and j in an event by:

$$y_{ij} = \frac{2\min(E_i^2, E_j^2)}{s}(1 - \cos\theta_{ij}) \tag{1}$$

where E_i and E_j are the energies of the two particles (or jets), θ_{ij} is the angle between them and s is the square of c.m. energy of the event. Jets or particles with $y_{ij} \leq y_{\rm cut}$ are combined into a single jet. This procedure is repeated untill all pairs i and j satisfy $y_{ij} > y_{\rm cut}$. The 2-jet events remained at the end of this process consist the 2-jet sample corresponding to the cut parameter $y_{\rm cut}$.

Since we are using Monte Carlo method, it is possible to study the 2-jet events both at parton level and at hadron level. Quark jet and antiquark jet can then be identified through matching the jets in these two levels using the following technique [5].

Firstly, identify the type of a single jet at parton level by checking every parton involved in this jet and giving a weight +1 to quark, -1 to antiquark and 0 to gluon. The sum of the weights is expected to yield +1 for quark jet and -1 for antiquark jet. The cases where the sum of weights in a jet is neither +1 nor -1, or the weights in the two jets of one event are both +1 or both -1 are neglected. This cut condition throw away about 1% of the total events.

Secondly, consider the configuration of the 2-jet event at parton and hadron levels. The directions of hadron jets might not be exactly that of the corresponding parton ones, since the effects of hadronization may change their directions.

The type of a hadron jet can be determined by checking the parton jet closest to it. The hadron jet closest to the parton level quark jet is considered as the quark jet and that closest to the parton level antiquark jet is identified as the antiquark jet. In this way, the quark jet subsample and the antiquark jet subsample are obtained.

3 Single-jet coordinate system

In the 2-jet events, the thrust (or sphericity) axis is supposed to be the direction in which the quark-antiquark move back to back with high momenta. Therefore, it is chosen as the longitudinal direction in the physics analysis. While when the single quark or antiquark jet is considered separately, the thrust axis is no longer appropriate to be considered as the longitudinal direction. The direction of the total momentum of the particles inside the jet should be taken as the longitudinal axis instead. Thus a jet coordinate system with the third axis pointing along the direction of jet momentum should be built up. The transformation from the lab system to the jet system is acomplished through the following two steps. First, rotate the lab frame o-xyz an angle φ around the z axis to form the o-$x'y'z'$ frame and then turn the latter an angle θ around y' axis to get the single-jet coordinate system o-$x''y''z''$, cf. Fig.1. This transformation can be expressed as:

$$\begin{pmatrix} x'' \\ y'' \\ z'' \end{pmatrix} = \begin{pmatrix} \cos\theta\cos\varphi & \cos\theta\sin\varphi & -\sin\theta \\ -\sin\varphi & \cos\varphi & 0 \\ \sin\theta\cos\varphi & \sin\theta\sin\varphi & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} \tag{2}$$

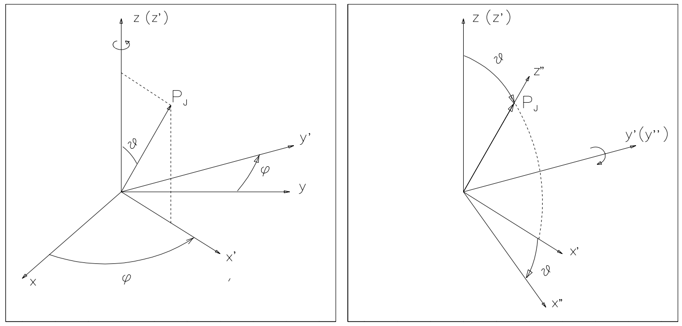

Fig.1 The transition from laboratory frame to quark (antiquark) jet frame

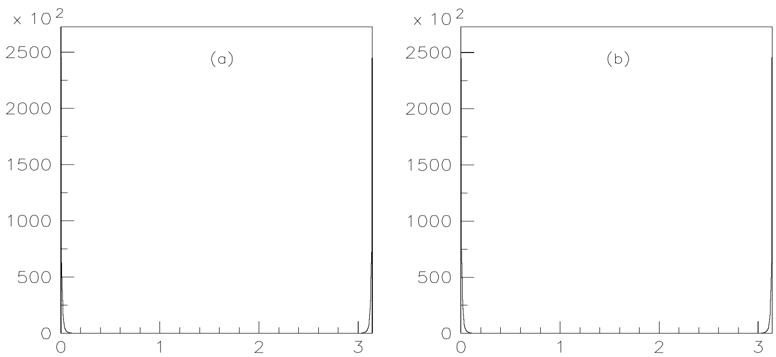

Fig.2 The distribution of the angle between the quark (antiquark) jet axis and the thrust axis. (a) quark jet (b) antiquark jet

The distribution of the angular difference between the direction of the quark (antiquark) jet axis and the thrust axis is shown in Fig.2. It can be seen from the figure that the jet axis is very close to the thrust axis.

4 Dynamical fluctuations

The dynamical fluctuations are characterized by the anomalous scaling of normalized factorial moments (NFM)[6]

$$F_q(M) = \frac{1}{M}\sum_{m=1}^{M} \frac{\langle n_m(n_m-1)\cdots(n_m-q+1)\rangle}{\langle n_m\rangle^q} \sim M^{\phi_q} \quad M \to \infty, \tag{3}$$

where a region Δ in one-, two-, or three-dimensional phase space is divided into M cells, n_m is the multiplicity in the mth cell, and $\langle\cdots\rangle$ indicates vertically averaging over the event sample, consisting of quark or antiquark jets.

As Ochs pointed out, the dynamical fluctuations occured in higher-dimentional (2D or 3D) phase space have the projection effect[7] on the fluctuations in lower-dimentional space causing the second-order 1D NFM goes to saturation by the rule [8]:

$$F_2^{(a)}(M_a) = A_a - B_a M_a^{-\gamma_a}, \qquad a = y, p_{\rm t}, \varphi \tag{4}$$

where the exponent γ_a describes the rate of going to saturation of the NFM in the direction a and is the most important characteristic for the higher-dimensional dynamical fluctuations. If the values of γ in two directions are equal, $\gamma_a = \gamma_b$, the fluctuations are isotropic in the a, b plane; while when $\gamma_a \neq \gamma_b$ the fluctuations are anisotropic in this plane [9]. The nature of the dynamical fluctuations (or that of the fractal) can be expressed in terms of the Hurst exponent H_{ab}, which can be obtained from the values of γ_a and γ_b as [8]

$$H_{ab} = \frac{1+\gamma_b}{1+\gamma_a} \tag{5}$$

The dynamical fluctuations are isotropic (self-similar fractal) when $H_{ab} = 1$, and anisotropic (self-affine fractal) when $H_{ab} \neq 1$ [9].

In our Monte Carlo simulation a total number of 2000000 events of e^+e^- collisions at 91.2 GeV are produced by JETSET 7.4 generator. Different y_{cut} are used to select the 2-jet events in the phase spase region $[0 < y < 5]$, $[0.1 < p_{\text{t}} < 3.0]$, $[0 < \varphi < 2\pi]$ and the second order 1D factorial moments $F_2(y)$, $F_2(p_{\text{t}})$, $F_2(\varphi)$ are calculated respectively for both quark and antiquark jets. In order to avoid the influence of a non-flat distribution of the variables y, p_{t} and φ on the investigation of the dynamical fluctuations, all variables are transformed into their corresponding cumulant forms [10].

$$x(y) = \frac{\int_{y_a}^{y} \rho(y)dy}{\int_{y_a}^{y_b} \rho(y)dy}, \qquad x(p_{\text{t}}) = \frac{\int_{p_{\text{ta}}}^{p_{\text{t}}} \rho(p_{\text{t}})\text{d}p_{\text{t}}}{\int_{p_{\text{ta}}}^{p_{\text{tb}}} \rho(p_{\text{t}})\text{d}p_{\text{t}}}, \qquad x(\varphi) = \frac{\int_{\varphi_a}^{\varphi} \rho(\varphi)d\varphi}{\int_{\varphi_a}^{\varphi_b} \rho(\varphi)d\varphi}. \tag{6}$$

Then the fitting to the saturated curves for these second order NFM's vs. the partition number $M = 1, 2, \ldots, 40$ in 1D phase space is carried out and the corresponding parameters $\gamma_y, \gamma_{p_{\text{t}}}$ and γ_φ are obtained. In order to eliminate the influence of the momentum conservation, the first (for $F_2(\varphi)$ the first three) point(s) are omitted when fitting the data to Eq.(4).

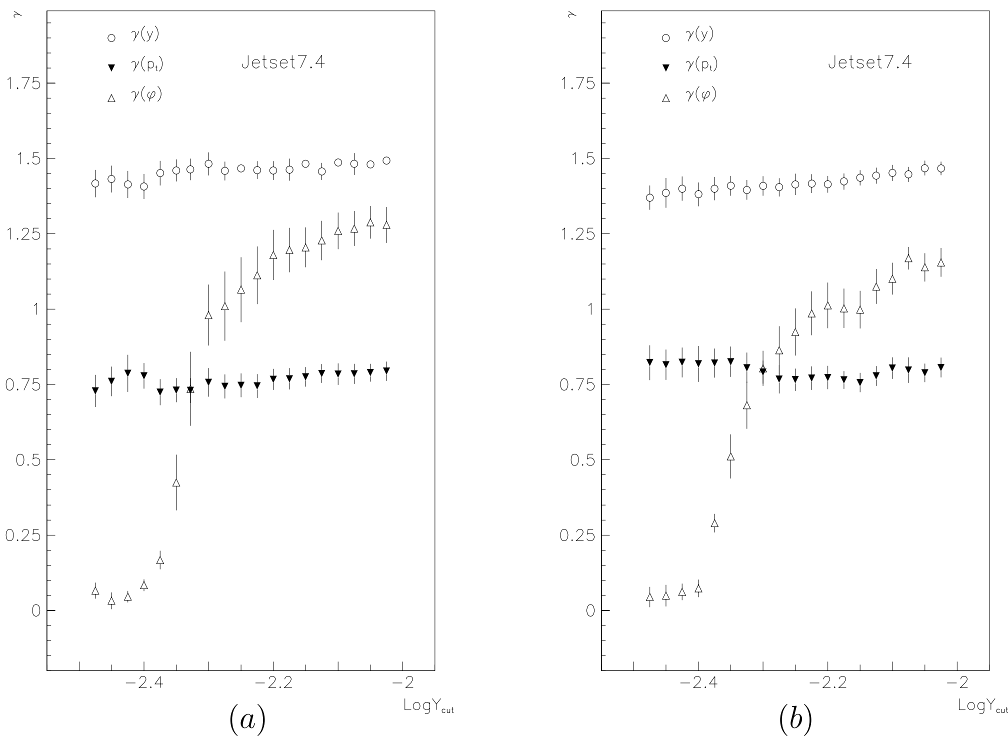

Fig.3 The parameter γ as function of $y_{\rm cut}$. (a) quark jets (b) antiquark jets

5. Results and discusion

The variation of the three γ_a ($a = y, p_{\rm t}, \varphi$) with $y_{\rm cut}$ are shown In Fig.3. From the figure some properties of the dynamical fluctuations inside the quark and antiquark jets can be extracted.

The most striking feature in Fig.3 is the variation of γ_φ with the parameter $y_{\rm cut}$. At small values of $y_{\rm cut}$, γ_φ lies at the bottom. When $y_{\rm cut}$ increases γ_φ increases rapidly and at a certain point γ_φ crosses over $\gamma_{p_{\rm t}}$, turning from $\gamma_\varphi < \gamma_{p_{\rm t}}$ to $\gamma_\varphi > \gamma_{p_{\rm t}}$. A distinct point called transition point [1], where $\gamma_\varphi = \gamma_{p_{\rm t}}$, exists in both quark- (Fig.3a) and antiquark (Fig.3b) jets with only slightly different values of $y_{\rm cut}$ — 0.0047 for quark jets and 0.0050 for antiquark jets.

At the transition point, the Hurst exponents are

$$H_{yp_{\rm t}} = \frac{1+\gamma_{p_{\rm t}}}{1+\gamma_y} = 0.70 \pm 0.03; \quad 0.74 \pm 0.03$$

$$H_{y\varphi} = \frac{1+\gamma_\varphi}{1+\gamma_y} = 0.70 \pm 0.06; \quad 0.75 \pm 0.04$$

$$H_{p_{\rm t}\varphi} = \frac{1+\gamma_\varphi}{1+\gamma_{p_{\rm t}}} = 1.00 \pm 0.10; \quad 1.01 \pm 0.06$$

The first values are for quark jets and the second ones for antiquark jets. These results indicate that at the transition point the dynamical fluctuations inside quark (antiquark) jet are anisotropic in the longitudinal-transverse planes, $(y, p_{\rm t})$ and (y, φ), and isotropic in the transverse plane $(p_{\rm t}, \varphi)$. This means that the quark and antiquark jets are circular in the transverse plane with respect to the dynamical fluctuations at the transition point and therefore they are called circular jets [1]. For these jets $H_{yp_{\rm t}} \neq 1$, $H_{y\varphi} \neq 1$ showing that the quark (antiquark) jet system is self-affine fractal[9]

In the Durham algorithm used in our calculation the relative transverse momentum $k_{\rm t}$ is related to $y_{\rm cut}$ as [12]

$$k_{\rm t} = \sqrt{y_{\rm cut}} \cdot \sqrt{s} \tag{7}$$

The value of $k_{\rm t}$ corresponding to the transition point is approximately 6.25 GeV for quark jet and 6.46 GeV for antiquark jet. The values of various parameters for both quark and antiquark jets at the transition point are listed in Table I

Table I Parameters γ, Hurst exponent H, cut-parameter $y_{\rm cut}$ and relative transverse momentum $k_{\rm t}$ at the transition point

subsample	$y_{\rm cut}$	γ_y	$\gamma_{p_{\rm t}}$	γ_φ	$H_{y\ pt}$	$H_{y\varphi}$	$H_{p_{\rm t}\varphi}$	$k_{\rm t}$ (GeV)
quark	0.0047	1.464	0.730	0.735	0.70	0.70	1.00	6.25
jet	±0.0003	±0.036	±0.041	±0.123	±0.03	±0.06	±0.10	±0.17
antiquark	0.0050	1.409	0.791	0.803	0.74	0.75	1.01	6.46
jet	±0.0009	±0.033	±0.039	±0.059	±0.03	±0.04	±0.06	±0.58

Having analyzed the dynamical fluctuations inside quark (antiquark) jets, it is interesting to discuss the following question. In high energy experiments some algorithm, e.g. the Durham algorithm, is used to select jets. In doing so, the jet sample obtained strongly depends on the cut parameter $y_{\rm cut}$. When the value of $y_{\rm cut}$ is chosen appropriately, the selected jet will be directly observable in the experiments as a bunch of particles. These directly observable jets are refered to as visible jets [1]. The question is: Is the circular jets determined by the properties of the dynamical fluctuations at the transition point consistent with the visible jets in experiments?

In order to see the relation between the circular jets determined by the transition point and the visible jets observed in experiments, let us notice the fact that it was in the energy region 17–30 GeV that a third jet (the gluon jet) was historically first observed in e^+e^- collisions [11]. In Fig.4's are shown the ratios R_3 of 3-jet events as function of the relative transverse momentum $k_{\rm t}$ at 5 different energies ranging from 11 to 91.2 GeV. The dashed vertical lines correspond to the transition point $k_{\rm t}$=6.25 GeV and $k_{\rm t}$= 6.46 GeV, respectively.

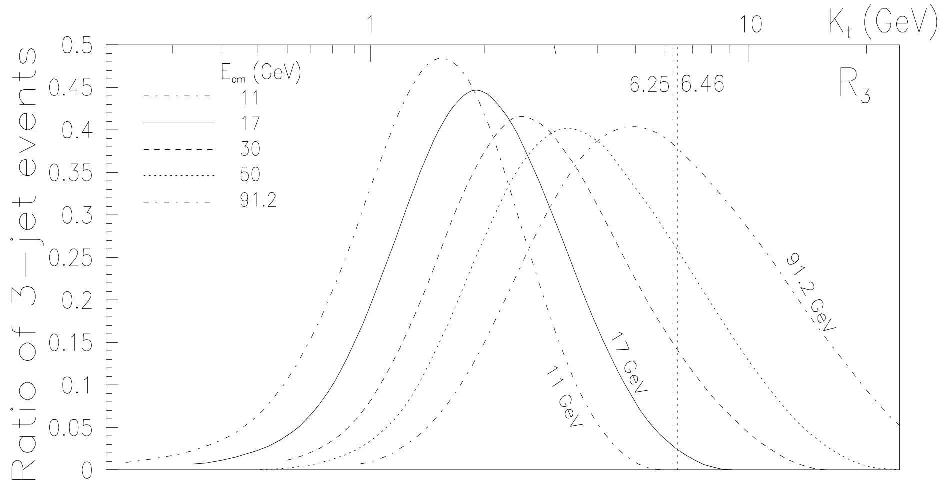

Fig.4 The ratio of R_3 as function of the relative transverse momentum k_{t}

It can be seen from the figure that for the circular jets defined by the transition point $k_{\mathrm{t}} = 6.25 \sim 6.46$ GeV (dashed lines in the figure), 3-jet events become noticeable just at $\sqrt{s} \approx 17$ GeV. This means that the third (gluon) jet first observed at $\sqrt{s} = 17$ GeV is consistent with being the circular jet.

4. Conclusion

In this paper we identify the hadronic quark- and antiquark-jets through matching them with the parton-level jets. Then we study the dynamical fluctuations inside the quark jets and the antiquark jets respectively using Monte Carlo simulation.

The second order 1D factorial Moments of the three variables $y, p_{\mathrm{t}}, \varphi$ are calculated. By fitting the results to the projection formula Eq.(4) the variation of the saturation exponents $\gamma_y, \gamma_{p_{\mathrm{t}}}$ and γ_φ with y_{cut} is obtained. A transition point exists in both the quark and the antiquark jets. At the transition point the dynamical fluctuations inside the quark jets and the antiquark jets, being similar to each other, are anisotropic in the longitudinal-transverse planes and isotropic in the transverse plane. These jets are refered to as circular jets.

The percentage R_3 of 3-jet events for the jets of different k_{t} have been calculated. It turns out that for the circular jets defined by the transition point $k_{\mathrm{t}} = 6.25 \sim 6.46$ GeV, the percentage R_3 of 3-jet events becomes noticeable just at $\sqrt{s} \approx 17$ GeV, where the 3-jet event was historically first observed. Therefore, the circular jets defined by the k_{t} at transition point is consistent to be the visible jets, which are directly observable in experiments.

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